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**RATE DEPENDENT CONSTITUTIVE
MODELS FOR FIBER REINFORCED
POLYMER COMPOSITES**

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Rate Dependent Constitutive Models for Fiber Reinforced Polymer Matrix Composites

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Summary

A literature survey has been conducted to assess the state-of-the-art in rate dependent constitutive models for continuous fiber reinforced polymer matrix composite (PMC) materials. Several recent models which include formulations for describing plasticity, viscoelasticity, viscoplasticity and rate-dependent phenomenon such as creep and stress relaxation are outlined and compared. When appropriate, these comparisons include brief descriptions of the mathematical formulations, the test procedures required for generating material constants and details of available data comparing test results to analytical predictions.

1 Introduction

Constitutive models to describe the rate dependent behavior of isotropic metallic materials have taken many forms over the last 4 decades. Examination of published constitutive models reveal that several different forms of viscoelasticity and viscoplasticity have been proposed and utilized by the mechanics community. During 1966, in an extensive article on viscoplasticity, Perzyna [1] provided descriptions of several rate dependent constitutive models and their capabilities. Further descriptions of rate dependent constitutive models for metallic materials can be found in Cristescu and Suliciu's

book on viscoplasticity [2] which was published in 1982. These two sources provide a good background on many of the accepted models used to describe rate dependent behavior in isotropic metallic materials. In addition to these sources, Lindholm et.al. [3] recently reviewed constitutive models for advanced metallic alloys used in high temperature applications.

The present survey paper is focused on advances in the application of rate dependent constitutive models to continuous fiber reinforced organic matrix composites. In order to review the state-of-the-art, emphasis will be given to recent viscoelastic and viscoplastic constitutive models for PMC materials.

In general, all of the models outlined in the references mentioned above were developed for use in describing rate dependent behavior in isotropic materials. In recent years, many researchers have relied on some of the models described in these sources to provide a foundation for developing rate dependent constitutive models for PMC materials. In many cases, this is an acceptable starting point due to the fact that PMC's may exhibit phenomenological behavior which appears similar to that seen in advanced metallics.

Despite some apparent similarities between observed behavior in metallics and PMC's, one of the primary gross physical differences between isotropic directionally independent metals and PMC's is that continuous fiber reinforced composites are anisotropic with well defined directional dependency. As a result of this material anisotropy and the laminated construction common to most PMC's, the mechanics of the material which contribute to the observed rate dependent behavior in PMC's may be quite different when compared to the mechanics of isotropic metallics. Consequently, the constitutive models used to describe rate dependent elastic and inelastic behavior may incorporate more material constants and include more terms than the general isotropic case. This increased complexity implies that test requirements for generating the necessary material constants will be different than those utilized in the isotropic models and that experimental procedures will be more involved. From an analytical viewpoint, the inclusion of material anisotropy implies that the constitutive models for PMC's will usually include the specific case of material isotropy as the most general application of the composite material rate dependent constitutive model.

It is generally recognized that most commercial fibers used in composites will exhibit linear elastic behavior up to very close to the ultimate failure point when loaded along their longitudinal axis. Conversely, many composite matrix materials may exhibit material nonlinearity and rate dependent

behavior when under load. The degree of this type of behavior will depend upon the molecular make-up of the matrix material, the operating environment, the load level and the matrix interaction with the fibers while under a general state of stress.

For many years, the primary types of composite materials of interest were based upon a thermoset epoxy matrix system. Generally, these composites were characterized as brittle and elastic. Therefore, many of the early rate dependent constitutive models for composites concentrated upon defining viscoelastic material behavior.

One early model of this type was formed by assuming time independent properties along the fiber direction for composites under isothermal conditions. Dillard et.al. [4,5] utilized this viscoelastic concept to describe the failure of graphite/epoxy laminates while undergoing creep. In other work, Schaffer and Adams [6] performed nonlinear viscoelastic analysis of unidirectional composites by using a form of Schapery's [7] integral constitutive equation. As part of this work, they formed a micromechanics model which allowed them to study the effects of the individual constituents on the laminate behavior. Their model was implemented via a finite element code. Material constants for the model were found by using uniaxial test specimens under constant temperature conditions. Additional information on measurement of viscoelastic parameters can be found in work performed by Kibler and Carter [9]. More recently, Mohan and Adams [8] extended this work to include investigations of the nonlinear viscoelastic behavior of neat epoxy resin and unidirectional graphite/epoxy specimens. The effects of temperature and moisture absorption were also investigated. Numerical procedures were developed to construct the viscoelastic coefficients needed to characterize creep-recovery behavior.

Within the last decade, many new types of polymer matrix composites, such as graphite/thermoplastics, have been developed for use in aircraft and spacecraft structures. Many of these new materials have been shown to display aspects of both material nonlinearity and strain rate dependency. Several new PMC materials such as APC-2 and RADEL have also been developed for extended use at elevated temperatures under conditions of static and cyclic mechanical loads. This condition of high operating temperature along with the increased toughness of the new PMC's has made it apparent that elastic, linear, rate-independent analysis procedures will not be sufficient for a complete characterization of the material behavior.

Viewing the PMC as a tough material has required constitutive models to be developed which utilize concepts of rate independent plasticity and rate dependent viscoplasticity in addition to rate dependent viscoelasticity. These types of models may include formulations for describing the behavior of laminates under monotonic and cyclic loads for both uniaxial and multi axial states of stress. Various formulations have also been proposed to describe phenomenon such as creep and stress relaxation.

The remainder of this paper will outline some of the more current constitutive models for advanced PMC's and provide a comparison of their analytical formulations and associated test methods. The intent of this review is to provide a starting point for literature searches of current research in rate dependent constitutive models for PMC's.

2 Constitutive Model Description

Three types of constitutive models will be discussed in the following sections. Rate independent elastic/plastic models, rate dependent viscoelastic models, and rate dependent viscoplastic models. In general, two approaches have been used in the development of mechanical material properties for the constitutive models. The first approach assumes that the fiber and matrix material can be viewed as separate constituents and their individual contributions to composite behavior can be addressed. The second approach uses a lamina level model to describe the phenomenological behavior of the individual plies in the composite. Both types of approaches are used in the constitutive models given in the following paragraphs.

2.1 Rate Independent Elastic/Plastic Behavior

Providing an accurate description of the rate independent elastic/plastic behavior can be an important aspect in the complete description of the rate dependent viscoelastic or viscoplastic behavior. With this in mind, two rate independent elastic/plastic models are described below.

In order to describe the observed plasticity in continuous fiber reinforced metal matrix composites, Bahei-El-Din and Dvorak [10,11] developed a micromechanics constitutive model based upon a continuum reinforced by cylindrical fibers of vanishingly small diameter. Their model relied upon a mi-

micromechanics description of the material and used volume fractions of the individual constituents to assign relative contributions of the fiber and matrix. This model may be considered an extension of Hill's[12,13,14] micromechanics constitutive model for composites. In Bahei-El-Din and Dvorak's model, the stress and strain increments were written as

$$d\sigma = C_f d\sigma_f + C_m d\sigma_m \quad (1)$$

$$d\epsilon = C_f d\epsilon_f + C_m d\epsilon_m \quad (2)$$

where the volume fractions of the fiber and matrix are given as C_f and C_m respectively. The constituent relations in the elastic and plastic range for the fiber and matrix constituents were given as

$$d\sigma_r = L_r d\epsilon_r \quad r = f(\text{fiber}), \quad m(\text{matrix}) \quad (3)$$

with L being the matrix of instantaneous local moduli.

It was assumed that the fibers behaved elastically while the matrix exhibited plasticity and obeyed a yield condition given by

$$f(B_{me}\sigma) = 0 \quad (4)$$

where B_{me} is a stress concentration factor indicating the contribution of the individual constituent. The yield surface itself is of a VonMises type and is allowed to harden kinematically. The flow rule is also specified and is associated with the hardening rule.

Comparison with experimental data in reference [11] indicated that the model provided a reasonable approximation of the observed behavior for a $[0/\pm 45]_s$ boron reinforced aluminum laminate subjected to two loading cycles. This is shown in figure 1. It was noted that the model should be used on materials with low to moderate fiber densities and stress states with low isotropic components. In addition, it was found that applications of the model to thermal problems may require further theoretical development. Although the model was applied to metal matrix composites, it appears likely that it may also be applied with some confidence to PMC's.

In a series of articles on orthotropic plasticity, Sun and his co workers[14-17] constructed a constitutive model for metal matrix and polymer matrix composites. The original form of the model was formulated by Kenaga et.al.[15] for the characterization of the nonlinear behavior of boron/aluminum

composites under a state of uniaxial tension. This model relied upon a yield function which required three material parameters and was quadratic in stresses. Analytical results from the model were compared to test data from off-axis tension tests and found to correlate well within the range of loading cases investigated.

This three parameter model was investigated further by Rizzi et. al. [16] for cases involving laminated composites in a nonuniform stress field. The constitutive model was incorporated into a finite element program by Leewood et. al. [17] and verified by comparison to test data from tapered specimens and rectangular specimens with a center hole. It was shown that the selection of the proper parameters in the uniaxial work hardening law resulted in a good comparison between test data and finite element predictions.

In the most recent form of the model by Sun and Chen [18], the model was simplified further by using a one parameter flow law. This model assumed that the total incremental strain can be decomposed into elastic and plastic tensor components given by

$$d\epsilon_{ij} = d\epsilon_{ij}^e + d\epsilon_{ij}^p \quad (5)$$

Each of these components can be used in the constitutive relationships

$$d\epsilon_i^e = S_{ij}^e d\sigma_j \quad i, j = 1 - 6 \quad \text{elastic} \quad (6)$$

$$d\epsilon_i^p = S_{ij}^p d\sigma_j \quad i, j = 1 - 6 \quad \text{plastic} \quad (7)$$

where S_{ij} represents the compliance matrix and $d\sigma$ is the increment in total stress.

The potential function used to describe the yield surface was found by assuming the composite exhibits linear elastic behavior in the fiber direction. In general, this function was given by

$$f(\sigma_{ij}) = 0 \quad (8)$$

with the Hill-type yield function for orthotropic materials [19] as a special case. The specific form of the yield function (f) used for fiber reinforced composites was given by

$$2f = \sigma_{22}^2 + 2a_{66}\sigma_{12}^2 \quad (9)$$

The use of one parameter, (a_{66}), in this function is significant in that only one type of test, uniaxial tension of an off-axis composite specimen, was required to quantify the parameter. The stress terms involved in this function are the ply transverse and shear stresses.

By using the associated flow rule, the incremental plastic strains were written in terms of the yield function. In addition, the increment in plastic work was defined using effective stress/strain quantities. Specifically, this was written as

$$dW^p = \bar{\sigma} d\bar{\epsilon}^p \quad (10)$$

where the effective stress was given by

$$\bar{\sigma} = \sqrt{3f(\sigma_{ij})} \quad (11)$$

and $d\bar{\epsilon}^p$ is the effective plastic strain increment. Use of these relations allowed for construction of complete constitutive relations to describe elastic/plastic behavior of laminates. Verification of the model by comparison to test data from polymer and metal matrix composites indicated that the chosen methods provided a reasonable approach to describing the observed nonlinear material behavior. Figures 2 and 3 show these results for off-axis stress/strain curves of boron/aluminum and graphite/epoxy composites at various fiber angles.

2.2 Viscoelastic Behavior

For describing rate dependent behavior, Schapery[20] has developed viscoelastic constitutive models for application to composites. In a recent report, Dan Jumbo, Harbert and Schapery[21] analyzed the strain rate effects and creep behavior of graphite/thermoplastic composites. In their analysis of this PMC, they examined both linear and nonlinear viscoelasticity solutions. For the linear viscoelastic solution, creep and recovery strains and the constant rate behavior are interrelated through a convolution integral

$$\epsilon = \int_{-\infty}^t D(t - \tau) \frac{d\sigma}{d\tau} d\tau \quad (12)$$

where $D(t)$ is the creep compliance. For a constant strain rate test, the time-averaged relaxation modulus is given by

$$E_s = \frac{1}{t} \int_0^t E(\tau) d\tau \quad (13)$$

and the recovery strain following the application of constant stress is

$$\epsilon = [t^n - (t - t_1)^n] D_o \sigma \quad (14)$$

where t_1 is the time under load and

$$D = D_o t^n \quad (15)$$

with D_o and n being positive constants.

The use of linear viscoelastic theory to analyze a laminate allowed for the ability to predict the creep modulus, $1/D$, and the exponent n in the creep strain. It was assumed that the modulus in the fiber direction and the principal Poisson's ratio were independent of rate and time. The relaxation modulus, in-plane shear modulus and the transverse modulus were all considered time dependent.

In dealing with the nonlinear viscoelastic behavior of laminates, a ply constitutive theory was used which was based upon an elastic/plastic model with constant elastic moduli modified for time dependence. Lamination theory was then employed to predict overall laminate response to uniaxial loads.

Uniaxial tension tests were run on rectangular specimens with varying amounts of 0° , $\pm 45^\circ$, and 90° plies. Constant rate and creep-recovery tests were performed. A comparison between test and theory showed a good agreement for most cases. However, some errors were observed in the prediction of creep in the nonlinear range during the creep-recovery tests.

For constant rate tension loading, figure 4 shows Dan Jumbo's comparison of test to theory for longitudinal stress/strain and Poisson's ratio of a $[0/\pm 45/90]$ laminate. Results from both the linear and nonlinear theory are shown in this figure. Examination of the results by the authors revealed that the proposed method was inaccurate at high stresses when dealing with the creep strains which occur just after loading. Despite this, the nonlinear theory appears to give an accurate representation of the complete axial stress/strain behavior.

In an analytical study of the thermo-viscoelastic behavior of composite materials, Lin and Hwang[22] developed a finite element model using a variational formulation. A finite difference scheme was also developed to solve the integral equations.

The constitutive model used by Lin and Hwang is for a linear viscoelastic

orthotropic material and is provided by the relation

$$\sigma_i(T, t) = \int_{-\infty}^t C_{ij}(T, t - \tau) \frac{\partial}{\partial \tau} (\epsilon_j(\tau) - \epsilon_j^*(\tau)) d\tau \quad (16)$$

This is a form of the hereditary integral given by Schapery [23] which is similar to equation 12 above. For this integral, σ_i is the stress, ϵ_j is the total strain and ϵ_j^* is the free thermal strain. The matrix C_{ij} is composed of the relaxation moduli which are functions of time (t) and temperature (T).

Lin and Hwang assumed that the composite material was thermo-rheologically simple and that the relaxation moduli could be found from a master curve by knowing a reference temperature and the temperature shift factor[24]. For use in the computational scheme, the relaxation moduli were given in terms of an exponential series

$$C_{ij}(t) = C_{ij,0} + \sum_{\omega=1}^{NT} C_{ij,\omega} e^{-t/\lambda_{ij,\omega}} \quad (17)$$

where NT is the number of terms in the series and the relaxation times are given by the constants $\lambda_{ij,\omega}$

Using their finite element model, the viscoelastic behavior of both notched and unnotched laminates were obtained. The effects of environmental load spectrums were also investigated. Material constants needed by the model were taken from the literature using test data from orthotropic lamina. The case of a $[\pm 45/0/90]_s$ layup under isothermal loads was compared to lamination theory results predicted by Tuttle and Brinson[25]. A comparison of the two methods showed good agreement. This comparison is given in figure 5. Additional test cases were run but not compared to test data or other analytical results.

2.3 Viscoplastic Behavior

In order to describe the observed rate dependent behavior of graphite fiber reinforced thermoplastic composites, Gates and Sun[26,27] developed an elastic/viscoplastic constitutive model for an orthotropic material. For their model, it was assumed that the rate dependent strain could be decomposed into an elastic and plastic component

$$\dot{\epsilon}_{ij} = \dot{\epsilon}_{ij}^e + \dot{\epsilon}_{ij}^p \quad (18)$$

where $\dot{\epsilon}$ implies a derivative with respect to time and e and p indicate elastic and plastic components, respectively.

The constitutive relation for the elastic terms was given by

$$\dot{\epsilon}_i^e = S_{ij}^e \dot{\sigma}_j \quad i, j = 1 - 6 \quad (19)$$

where S_{ij}^e is the compliance matrix of elastic constants. Similarly, the viscoplastic constitutive relationship was written as

$$\dot{\epsilon}_i^p = S_{ij}^{vp} \dot{\sigma}_j \quad i, j = 1 - 6 \quad (20)$$

where S_{ij}^{vp} is the viscoplastic compliance matrix found by using a form of the associated flow rule

$$\dot{\epsilon}_{ij}^p = \frac{\partial f}{\partial \sigma_{ij}} \dot{\lambda}' \quad (21)$$

The quantity (f) in this expression is the potential function which describes the rate dependent loading surface, while $\dot{\lambda}'$ is a proportionality factor used to bring in the time rate of change of stress. The potential function, given by equation 9, accounts for material anisotropy and is formed by assuming elastic behavior along the fiber direction and plane stress conditions.

The dynamic loading surface was assumed to be everywhere parallel to the static yield surface with the plastic strain rate vector acting in a direction normal to it. Consequently, the time independent elastic/plastic yield surface and the rate dependent dynamic loading surface were represented by the same potential function.

The proportionality factor $\dot{\lambda}'$ in equation 21 was given more explicitly by the expressions

$$\dot{\lambda}' = \gamma \langle \Phi(H) \rangle \quad (22)$$

$$\gamma = \frac{3}{2} \bar{\sigma} \quad (23)$$

where $\bar{\sigma}$ is the effective stress and $\Phi(H)$ was given by

$$\langle \Phi(H) \rangle = \begin{cases} \Phi(H) & \text{if } H > 0 \text{ (loading)} \\ 0 & \text{if } H \leq 0 \text{ (unloading)} \end{cases} \quad (24)$$

The scalar quantity H is the "overstress" and was defined by

$$H = [\bar{\sigma} - \bar{\sigma}^*] \quad (25)$$

where $\bar{\sigma}$ and $\bar{\sigma}^*$ are the effective rate dependent and effective quasistatic stress, respectively. The use of effective stress quantities allows for the dependence of the state of stress on the angle between the load and fiber directions and can be represented in a general sense by equation 11. The quasistatic stress is assumed to be rate independent and is found by using the incremental elastic/plastic expressions of Sun and Chen[18].

The concept of overstress and its relationship to viscoplastic strain in isotropic metallics has been attributed to Malvern[28] and his work on high strain rate conditions during wave propagation. Additional references to overstress and its use in constructing viscoplastic models in metallics can be found in the work of Eisenberg and Yen[29,30].

Gates and Sun [26,27] adopted the concept of overstress but changed it to relate to effective stress in PMC's. In turn, the effective stress was related to the combined state of stress which existed in orthotropic lamina under a state of off-axis uniaxial loading. Use of effective stress and strain along with the appropriate potential function allowed for the generation of master curves for elastic/plastic and elastic/viscoplastic material constants. Overstress, and hence, the viscoplastic parameters, were found by performing multiple relaxation events during a single test and measuring the change in stress levels over time.

Material constants and related functions needed by this model were found by performing uniaxial tension tests on off-axis laminates under isothermal conditions. By collapsing this data into master curves, five experimentally derived parameters could be incorporated into the constitutive model. Comparison of the predicted behavior to experimental data showed a good correspondence for both variable strain rate loading and stress relaxation of off-axis laminates. The typical measured and predicted stress/strain time dependent behavior of an off-axis laminate is shown in figure 6. Additional information on the use of this type of model and its comparison to test data is given in reference [31].

Another analytical model developed for describing the rate dependent behavior of PMC's was developed by Ha and Springer[32]. Their model utilized both viscoplastic and viscoelastic constitutive relationships to calculate the rate dependent response of composite laminates. These relationships were developed for individual unidirectional lamina and then used in a modified form of lamination theory to predict the response of a multidirectional laminate.

For their model, it was assumed that the mechanical strain was composed of three components

$$\epsilon_i^{mech} = \epsilon_i^{ne} + \epsilon_i^{ve} + \epsilon_i^{vp} \quad (26)$$

where the three terms represent nonlinear elastic, viscoelastic and viscoplastic strains, respectively, and the subscript i indicates material direction. Additional assumptions were that the total strain was to consist of the sum of the mechanical and thermal strains, and the ply properties in the fiber direction were to be independent of time.

The nonlinear elastic strain ϵ_i^{ne} , as given by Ha and Springer[33], for a ply is

$$\epsilon_i^{ne} = \frac{\sigma_i}{E_i^l} \left[1 + \left(\frac{\sigma_i}{\bar{\sigma}_i^l} \right)^2 \right] \quad (27)$$

where l indicates the type of loading and E and $\bar{\sigma}$ are temperature dependent parameters.

The viscoelastic strain component ϵ_i^{ve} was assumed to behave nonlinearly and exhibit temperature dependency. A form of the Schapery-Lou [34] integral expression, similar to equation 12, was given by

$$\epsilon_i^{ve} = g_{i1} \int_0^t K_i \frac{\partial g_{i2} \sigma_i}{\partial \tau} d\tau \quad (28)$$

where K_i is the time dependent creep compliance function which was given as the summation of an arbitrary number of exponential terms. The additional parameters g_{i1} and g_{i2} depend upon stress and temperature. The total number of experimentally derived material constants needed for this viscoelastic component is 23 for shear and transverse loading combined.

For a single ply, an associated flow law was written to relate viscoplastic strain rate and stress. The assumed form was

$$\dot{\epsilon}_i^{vp} = \frac{1}{\eta_i} \phi_i \frac{\partial f}{\partial \sigma_i^*} \quad (29)$$

where η is a constant and the potential function f was given by

$$f = \sqrt{\left(\frac{\sigma_y}{X_y^l} \right)^2 + \left(\frac{\sigma_s}{X_s} \right)^2} \quad (30)$$

The stress term in equation 29 is dimensionless and was given by

$$\sigma_i^* = \frac{\sigma_i}{X_i^l} \quad (31)$$

where X_i^l is the ply strength in the i direction.

The function ϕ_i in equation 29 is related to plastic strain rate and was assumed to be given by a form of overstress function written in the form of equation 24 with the overstress given by a power law

$$H = (\dot{\sigma} - \dot{\sigma}_i^{cr})^b \quad (32)$$

where the quantity $\dot{\sigma}_i^{cr}$ was allowed to vary with temperature and strain. In all, nine experimentally derived constants were required for the viscoplastic component assuming combined transverse and shear loads.

To predict the rate dependent behavior in laminated composites, a numerical solution routine for a modified laminated plate theory was developed. Experimental tests were run on graphite/epoxy specimens under four point bending. These tests were used to obtain the necessary material constants and to validate the model. Several layups, including multiple ply unidirectional, $[\pm 45]$ and $[\pm 60 / \pm 30]$ laminates were used. A comparison between test and theory indicated that the analytical model gave good predictions for creep behavior under various thermal and load histories. Figure 7 shows representative results for creep recovery tests for a $[\pm 45]_4$ layup under constant applied load and time varying temperature.

Sutcu and Krempf [35] and Krempf and Hong [36] also utilized the overstress concept to predict viscoplastic behavior in composites. Their model was applied to metal matrix composites and was formulated assuming that the total strain rate was composed of an elastic and inelastic component similar to equation 18 above. The formulation required no definition of yield surfaces and, consequently, loading and unloading conditions were not specified.

A modified form of laminate theory by Krempf and Hong assumed that total strain and strain rates were constant through the thickness and the stress rate was given by an average value. Material constants for the model were found from off-axis tests of unidirectional material. It was found that the most complete form of the model required 17 material constants and two experimentally derived functions. Stress relaxation and creep were investigated for both tension and compression loading; however, comparisons

to test data were given only for off-axis tension loading of unidirectional laminates. These comparisons indicated a reasonably accurate predictive capability. All of the cases explored in the analytical investigations were for isothermal conditions.

Two additional viscoplastic models which were developed for use on rate dependent anisotropic materials should be mentioned. These models were applied to metal matrix composites; however, they may have some application to PMC materials.

The first model was developed by Robinson and Duffy [37] and assumes the composite acts as a continuum with local transverse anisotropy. A macroscopic viewpoint of the material relationships was taken and a potential function, flow law and evolutionary law were developed and related to the material parameters. It was noted that some of the required parameters were dependent upon the volume ratio of the constituents. Material constants required by the model were found by performing tension/torsion tests on longitudinal and circumferential reinforced tubes. For the model, two parameters and one function were needed to relate the degree of anisotropy and threshold strength while four parameters and one function were needed for the viscoplastic components. Analytical predictions were made for specimens under several different states of stress. No comparison was given between test data and predicted values.

The other viscoplastic constitutive model which was applied to metal matrix composites was formulated by Aboudi [38] and assumes a microstructure based continuum. The analysis was performed by looking at a representative cell and assuming both the fiber and matrix to behave as elastic-plastic work hardening materials. In his derivations, Aboudi provided formulations for the displacement continuity conditions, the equations of motion for the continuum and the relevant boundary conditions. The constitutive equations used to describe the stress/strain relationships were based upon work by Bodner and Partom [39]. These relations have been described as a unified type of model which do not consider loading and unloading conditions separately. A comparison between test and theory was not made; however, analytical results for several different cases were considered and compared.

3 Model Capabilities

A summary comparison of four rate dependent constitutive models for PMC's is given in table 1. As was outlined above and is shown in table 1, all of the models share some similar capabilities and exhibit some unique differences. A comparison of the type of model indicates that selection of the analytical foundation is dependent upon which type of material behavior is of interest to the researcher. To account for rate-dependent behavior such as creep, creep-recovery, stress relaxation and aging, a combination of both viscoelasticity and viscoplasticity may be needed. In addition, use of concepts such as overstress to develop the viscoplastic strains may require a firm definition of rate independent elastic/plastic behavior.

Important considerations when examining a model are; assessment of the accuracy of the model when compared to actual experimental data, and complete explanations of methods needed to arrive at the necessary material constants. Figures 4-7 indicate the degree of accuracy of the selected models when compared to specific test cases. With respect to these figures, two criteria need to be kept in mind when comparing predicted values to experimental data. One, what is the relative error associated with the predictions for any point of interest? Two, does the model capture the correct overall form of the data during complex events such as loading and unloading, creep and relaxation? The first criteria of relative error may be of less important since sources of errors can usually be attributed to inadequate procedures in testing, data reduction or numerical implementation of the solution. This implies that the second criteria, capturing the correct form of the data, is of primary importance when assessing predictive capabilities. In particular, this criteria may be applied when it becomes necessary to choose a model to predict material behavior in actual structures. In any case, whatever criteria is selected, it is recommended that a complete investigation of the data presented in the references be made before judging the overall accuracy of the analytical model.

Development of experimental methods to generate test data and provision of a scheme for reducing it to the required material constants and empirical functions is also of importance when judging the applicability of any particular model. Table 1 indicates which models address these issues. Specific information on the simplicity and reproducibility of their respective test methods and data reduction schemes should be obtained directly from

Source of Model	Type of Model	Applied to			Experimental	
		Elev. Temp.	Creep	Relaxation	Comparison	Methods
DanJumbo ²¹	<i>VE</i>	No	Yes	No	Yes	Yes
Lin ²²	<i>VE</i>	Yes	Yes	Yes	No	No
Gates ²⁶	<i>VP</i>	Yes	No	Yes	Yes	Yes
Ha ³²	<i>VE/VP</i>	Yes	Yes	No	Yes	Yes

Table 1: Comparison of Selected Rate Dependent Constitutive Models, Key; *VE* \Rightarrow Viscoelastic, *VP* \Rightarrow Viscoplastic

the references themselves. The number of material constants required by any given model gives some indication of the complexity of the associated test techniques. However, a direct comparison of required constants for different models is not available since several of the models have constants which may be assumed or ignored for special case applications.

4 Further Research

This report compares some of the most recent constitutive models for rate dependent behavior in fiber reinforced composites. Particular emphasis has been placed upon those models which were applied to polymer matrix composites. These PMC constitutive models incorporate concepts of elasticity, plasticity, viscoelasticity and viscoplasticity. Important aspects of each model have been outlined and comparisons were made between analytical formulations, experimental methods and predictive capabilities.

Allowing for the fact that these models represent the state-of-the-art in rate dependent constitutive behavior for PMC's, several areas which require further research can be identified. Some of the more important future research topics are cyclic mechanical and thermal loading and associated frequency effects, behavior under repeated tension/compression loading, effects of material hybridization, material selection and processing, combined environmental effects, response to biaxial or triaxial stress states, fracture and damage growth. Many of these issues may need to be addressed and scrutinized before the proposed constitutive models can be fully accepted and utilized. Despite this, it is felt that as the uses of new advanced PMC's increase and the operating conditions for these materials become more severe,

analytical and experimental investigations on the types of constitutive models outlined in this report will be found to be invaluable in furthering our knowledge and capabilities.

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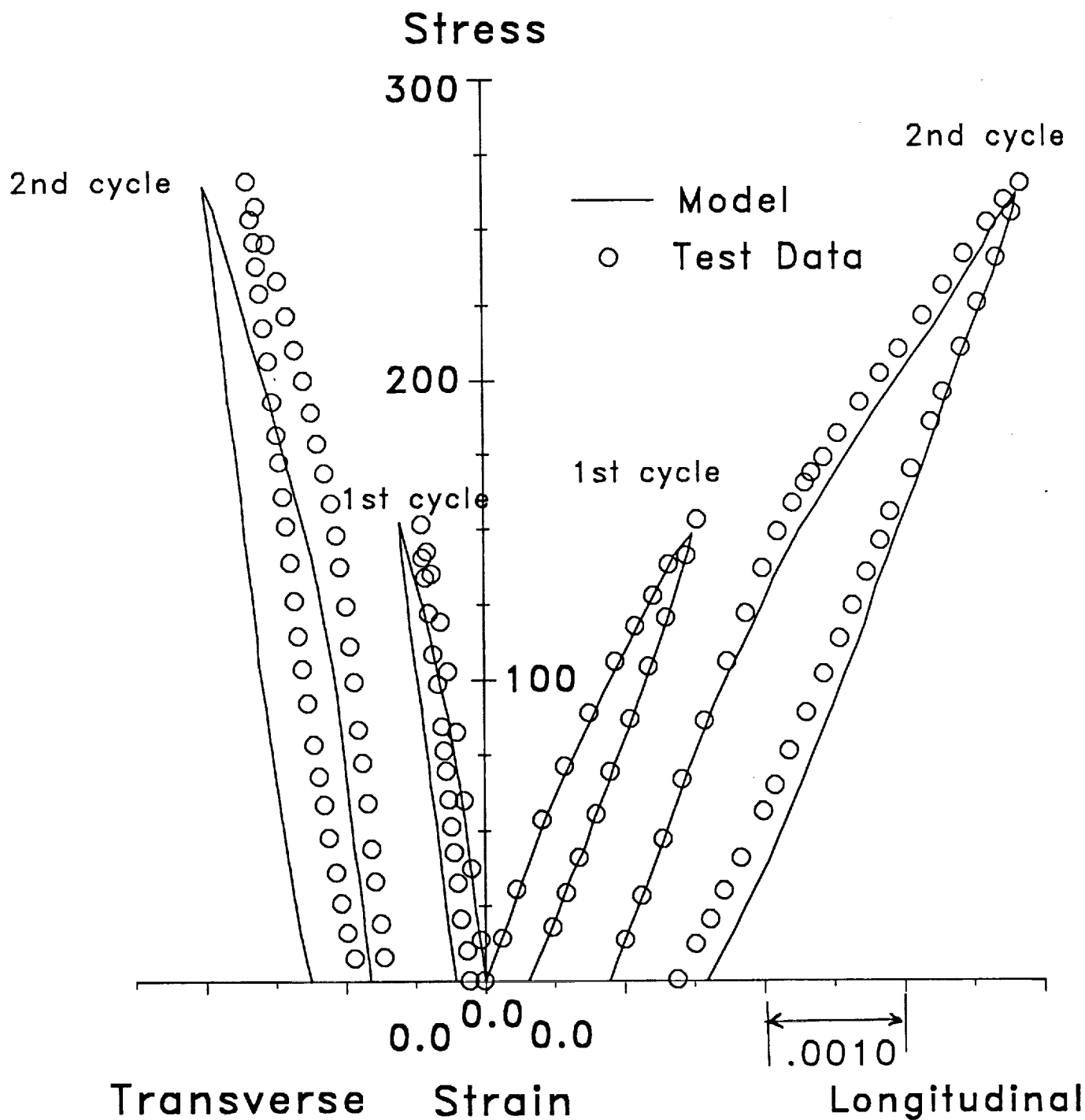


Figure 1: Comparison of calculated and measured stress-strain curves for a $[0/\pm 45]_s$ B-AL plate subjected to two loading cycles[11].

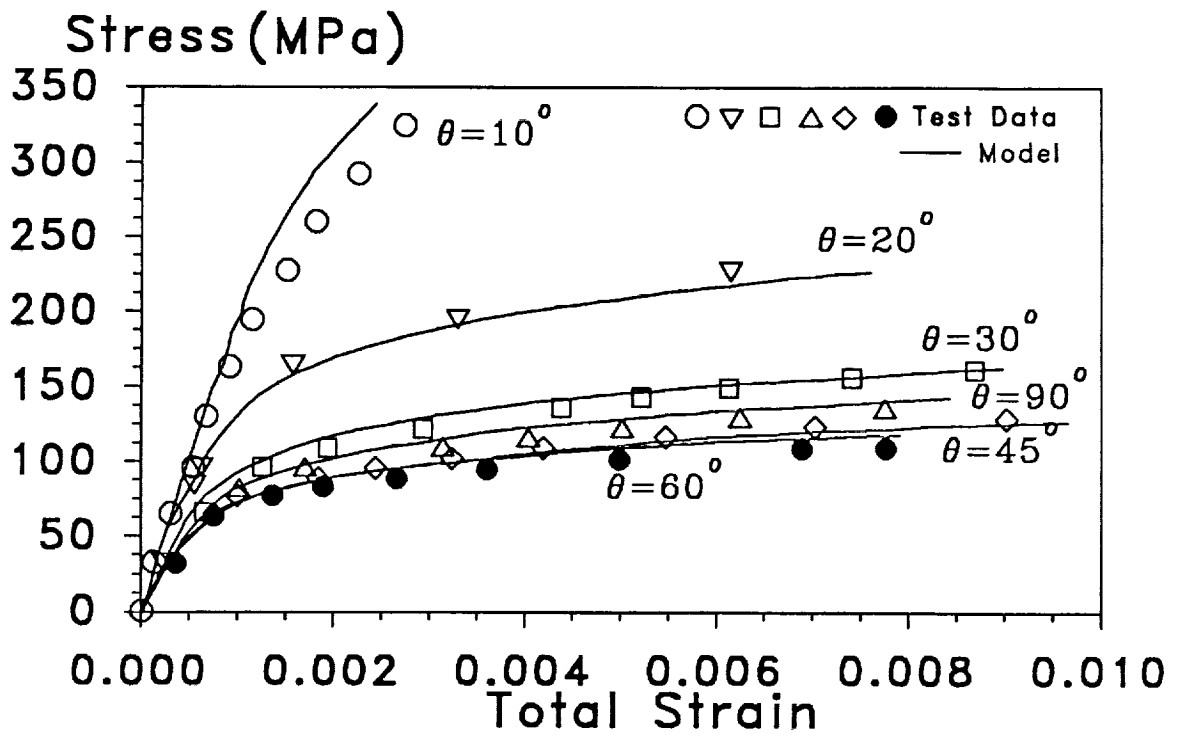


Figure 2: Comparison of calculated and measured off-axis stress-strain curves for B-AL material[18].

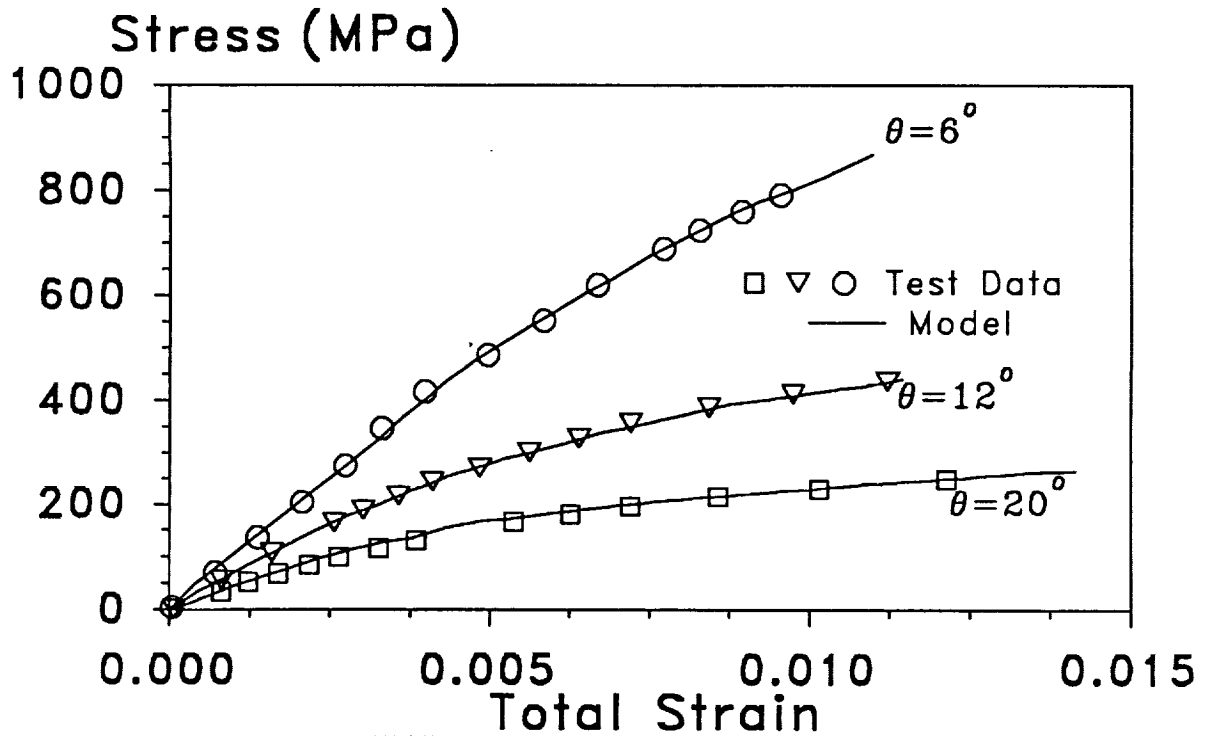


Figure 3: Comparison of calculated and measured off-axis stress-strain curves for graphite/epoxy material[18].

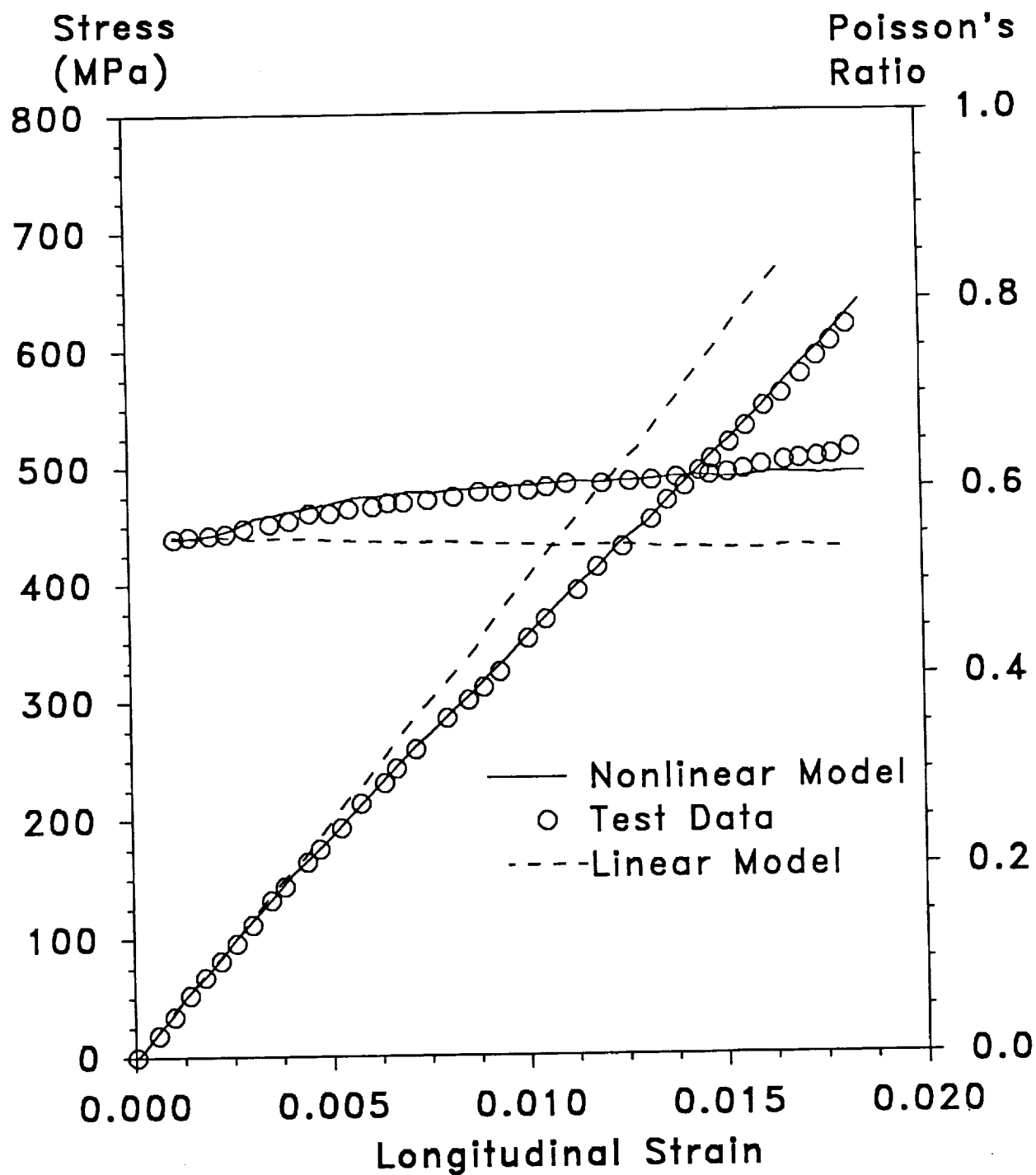


Figure 4: Comparison of calculated and measured stress-strain and Poisson's ratio curves for a $[0/\pm 45/90]$ graphite-PEEK material[21].

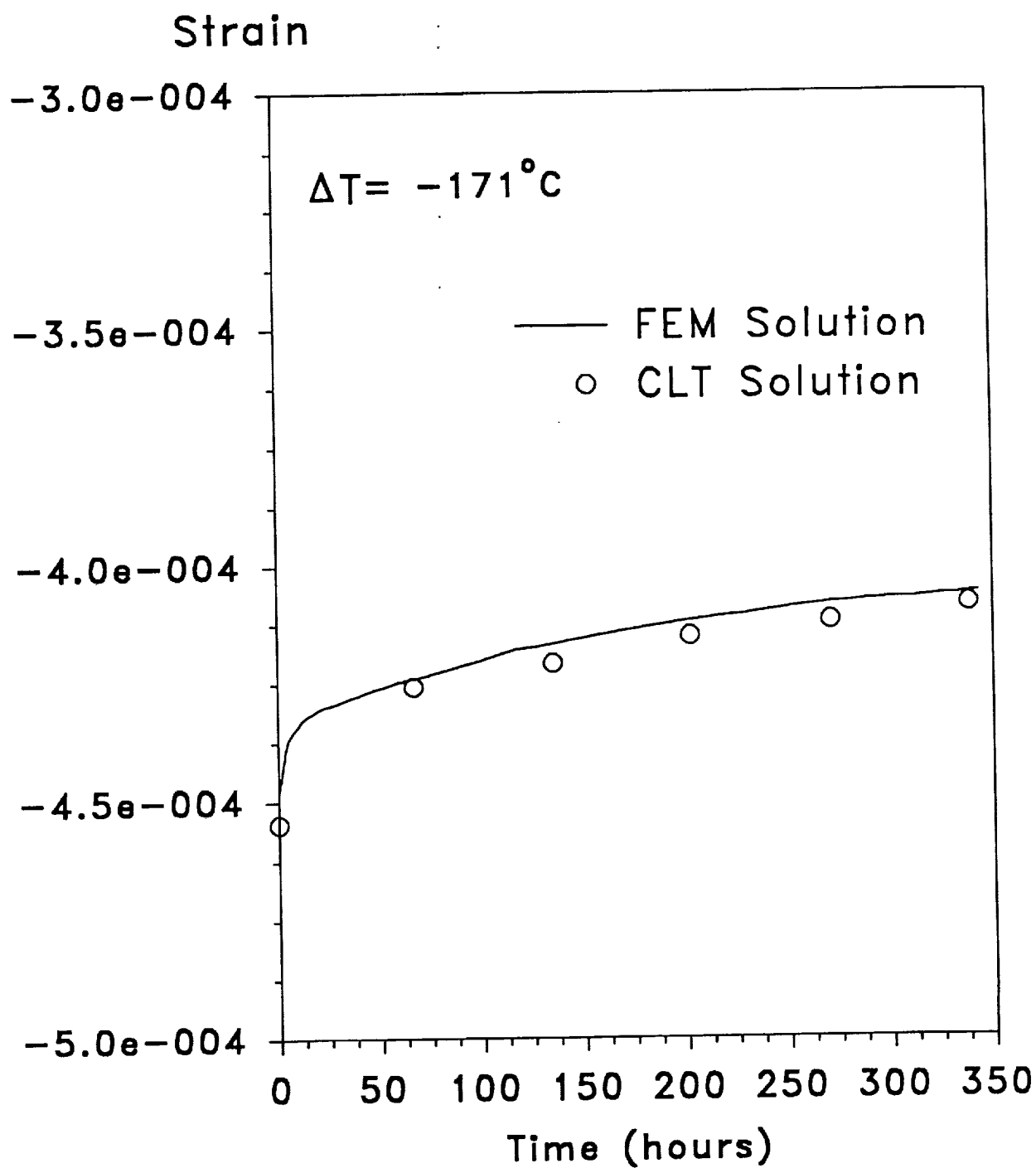


Figure 5: Comparison of calculated and classical lamination theory stress-strain curves for a $[\pm 45/0/90]$, graphite/epoxy material under thermal load[22].

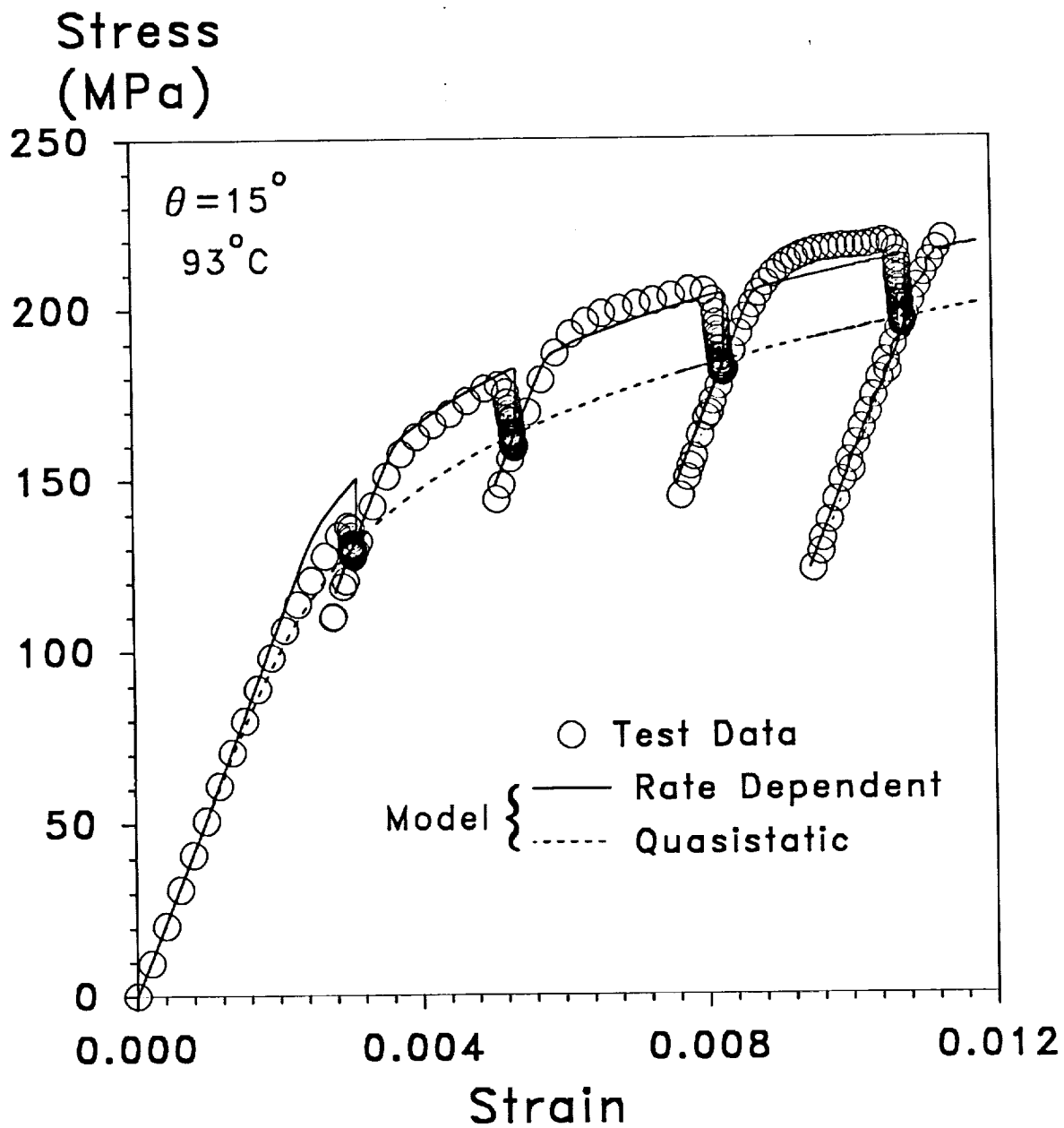


Figure 6: Comparison of calculated and measured stress-strain curves for an 15° off-axis graphite/PEEK specimen[27].

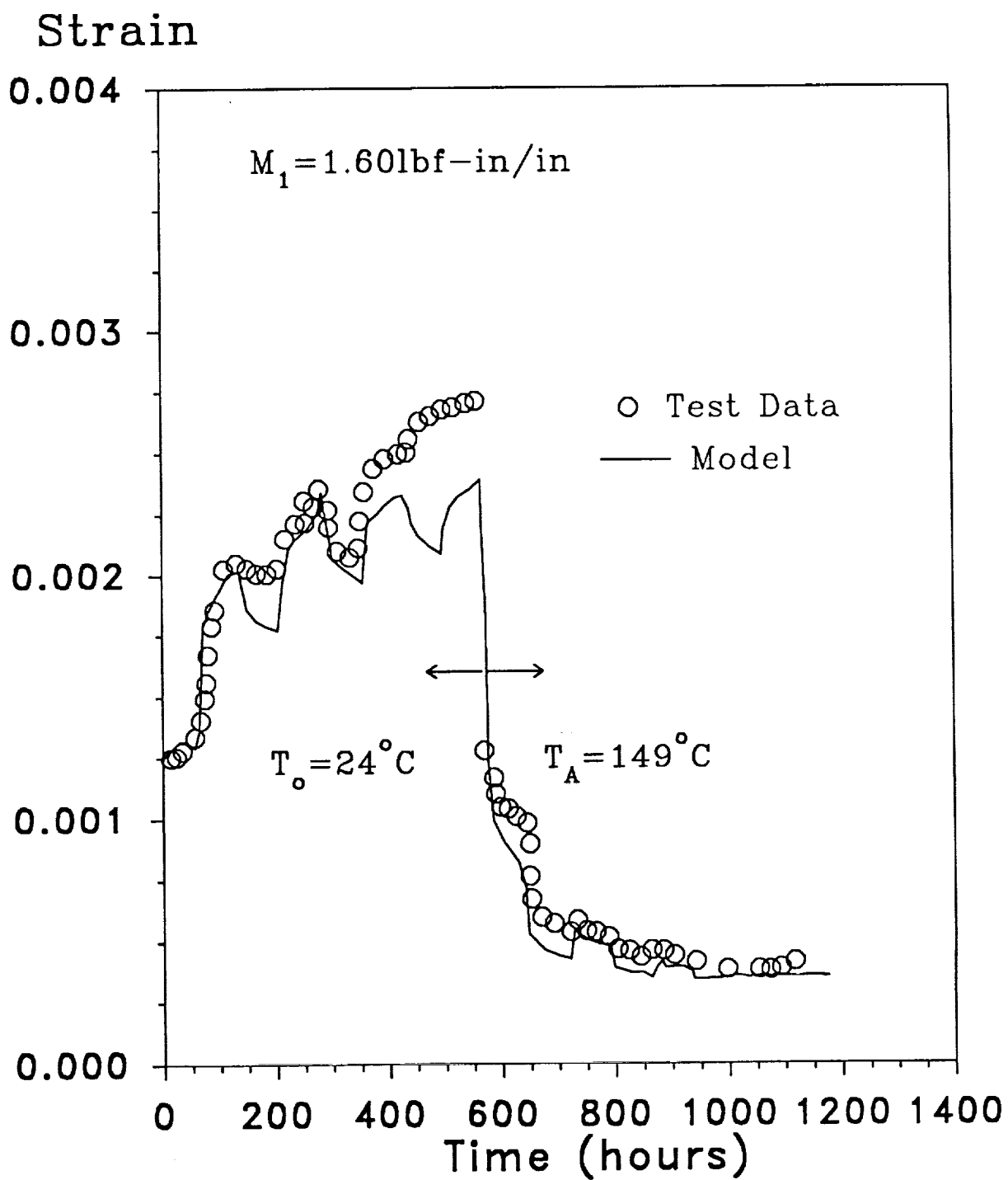


Figure 7: Comparison of calculated and measured strain-time curves for a $[\pm 45]_4$ graphite/epoxy material[32].



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